

# HOSSAM GHANEM

## (5) 7.3 The Logarithmic Differentiation

Differential :

$\frac{d}{dx} \log_a(x) = \frac{1}{\ln a} \cdot \frac{1}{x}$	$\frac{d}{dx} \log_9(x) = \frac{1}{\ln 9} \cdot \frac{1}{x}$
$\frac{d}{dx} \ln x = \frac{1}{x}$	$\frac{d}{dx} \ln(x - 5) = \frac{1}{x - 5}$

Logarithmic Differentiation:

Guidelines	خطوات الحل	Example
		Let $f(x) = x^x$ Find $f'(x)$
(1) Let $y = f(x)$	(1) ضع الدالة على الصورة $y = f(x)$	(1) Let $y = x^x$
(2) $\ln y = \ln f(x)$	(2) خذ $\ln$ الطرفين	(2) $\ln y = \ln x^x$
(3) simplify	(3) استخدم القوانين السابقة للتبسيط	(3) $\ln y = x \ln x$
(4) $\frac{d}{dx} \ln y = \frac{d}{dx} \ln f(x)$ $\frac{1}{y} y' = \frac{d}{dx} \ln f(x)$	(4) أشتق ضمناً	(4) $\frac{d}{dx} \ln y = \frac{d}{dx} x \ln x$ $\frac{1}{y} y' = \ln x + x \cdot \frac{1}{x}$
(5) $y' = y \frac{d}{dx} \ln f(x)$ $y' = f(x) \frac{d}{dx} \ln f(x)$	(5) أعد صياغة الناتج على الصورة الموضحة	(5) $y' = y (\ln x + 1)$ $f'(x) = x^x (\ln x + 1)$

Example 1 IF  $f(x) = \ln[(5x - 7)^4(2x + 3)^3]$  Find  $f'(x)$

Solution

$$f(x) = \ln[(5x - 7)^4(2x + 3)^3]$$

$$f(x) = 4 \ln(5x - 7) + 3 \ln(2x + 3)$$

$$\begin{aligned} f'(x) &= \frac{4(5)}{5x - 7} + \frac{3(2)}{2x + 3} \\ &= \frac{20}{5x - 7} + \frac{6}{2x + 3} \end{aligned}$$



**Example 2** Find  $\frac{dy}{dx}$  if  $y = \ln \tan^3(3x) + \cos(\ln 2x)$  9 October 1998

**Solution**

$$\frac{dy}{dx} = \frac{1}{\tan^3 3x} \cdot 3 \tan^2 3x \cdot 3 - \sin(\ln 2x) \cdot \frac{1}{2x} \cdot 2$$

$$\frac{dy}{dx} = \frac{9}{\tan 3x} - \frac{1}{x} \sin(\ln 2x) = 9 \cot 3x - \frac{1}{x} \sin(\ln 2x)$$

**Example 3** IF  $y = (x+1)^2(x+2)^3(x^2+3)^4$  Find  $y'$

**Solution**

$$\ln y = 2 \ln(x+1) + 3 \ln(x+2) + 4 \ln(x^2+3)$$

$$\frac{y'}{y} = \frac{2}{x+1} + \frac{3}{x+2} + \frac{4(2x)}{x^2+3}$$

$$y' = (x+1)^2(x+2)^3(x^2+3)^4 \left[ \frac{2}{x+1} + \frac{3}{x+2} + \frac{8x}{x^2+3} \right]$$

**Example 4** IF  $y = \sqrt{(3x^2+1)\sqrt{6x-7}}$  Find  $y'$

**Solution**

$$\ln y = \frac{1}{2} \ln(3x^2+1) + \frac{1}{4} \ln(6x-7)$$

$$\frac{y'}{y} = \frac{6x}{2(3x^2+1)} + \frac{6}{4(6x-7)}$$

$$y' = \sqrt{(3x^2+1)\sqrt{6x-7}} \left[ \frac{3x}{3x^2+1} + \frac{3}{2(6x-7)} \right]$$

**Example 5** Find  $dy/dx$  if  $y = \sqrt{\frac{x^2(2x-1)^3}{(x+5)^2}}$  18 July 2005 A

**Solution**

$$\ln y = \frac{1}{2} \cdot 2 \ln x + \frac{1}{2} \cdot 3 \ln(2x-1) - \frac{1}{2} \cdot 2 \ln(x+5)$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{3}{2} \cdot \frac{2}{2x-1} - \frac{1}{x+5}$$

$$\frac{dy}{dx} = \sqrt{\frac{x^2(2x-1)^3}{(x+5)^2}} \left( \frac{1}{x} + \frac{3}{2x-1} - \frac{1}{x+5} \right)$$



**Example 6**Use logarithmic differentiation to find  $\frac{dy}{dx}$  if 29 July 2009 A

$$y = \frac{x \exp(x^2)}{(e^{3x} + 1)(\sin^2 x)\sqrt{x + 1}}$$

**Solution**

$$\ln y = \ln x + x^2 - \ln(e^{3x} + 1) - 2 \ln|\sin x| - \frac{1}{2} \ln(x + 1)$$

$$\frac{y'}{y} = \frac{1}{x} + 2x - \frac{3e^{3x}}{e^{3x} + 1} - \frac{2 \cos x}{\sin x} - \frac{1}{2(x + 1)}$$

$$y' = \frac{x e^{x^2}}{(e^{3x} + 1)(\sin^2 x)\sqrt{x + 1}} \left( \frac{1}{x} + 2x - \frac{3e^{3x}}{e^{3x} + 1} - 2 \cot x - \frac{1}{2(x + 1)} \right)$$

**Example 7**Find  $\frac{dy}{dx}$  if  $y = \sqrt{\frac{\sec x \tan x}{\sqrt{\ln x + 2^x}}}$  6 March 1997**Solution**

$$\ln y = \frac{-1}{2} \ln(\cos x) + \frac{1}{2} \ln(\tan x) + \frac{1}{4} \ln(\ln x + 2^x)$$

$$\frac{y'}{y} = \frac{-1}{2} \cdot \frac{-\sin x}{\cos x} + \frac{\sec^2 x}{2 \tan x} + \frac{\frac{1}{x} + 2^x \ln 2}{4(\ln x + 2^x)}$$

$$y' = \sqrt{\frac{\sec x \tan x}{\sqrt{\ln x + 2^x}}} \left( \frac{1}{2} \cdot \tan x + \frac{1}{\sin 2x} + \frac{1 + 2^x x \ln 2}{4x(\ln x + 2^x)} \right)$$

**Example 8**Use logarithmic differentiation to find  $y'$  if 11 October 1999

$$y = \frac{(x + \log_5 x)^2 (x^3 + 2x)^9}{\sqrt[3]{x^3 + 1}}$$

**Solution**

$$\ln y = 2 \ln|x + \log_5 x| + 9 \ln(x^3 + 2x) - \frac{1}{3} \ln(x^3 + 1)$$

$$\frac{y'}{y} = \frac{2}{x + \log_5 x} \left( 1 + \frac{1}{x} \cdot \frac{1}{\ln 5} \right) + \frac{9(3x^2 + 2)}{x^3 + 2x} - \frac{3x^2}{3(x^3 + 1)}$$

$$y' = \frac{(x + \log_5 x)^2 (x^3 + 2x)^9}{\sqrt[3]{x^3 + 1}} \left( \frac{2}{x + \log_5 x} \left( 1 + \frac{1}{x \ln 5} \right) + \frac{9(3x^2 + 2)}{x^3 + 2x} - \frac{x^2}{(x^3 + 1)} \right)$$

**Example 9**Find  $\frac{dy}{dx}$  if  $y = (\log_5(\ln x^2))^{\tan x}$ 

9 October 1998

**Solution**

$$\ln y = \tan x \log_5(2 \ln|x|)$$

$$\frac{y'}{y} = \sec^2 x \log_5(2 \ln|x|) + \tan x \cdot \frac{1}{2 \ln|x|} \cdot \frac{1}{\ln 5} \cdot \frac{2}{x}$$

$$y' = (\log_5(\ln x^2))^{\tan x} \left( \sec^2 x \log_5(\ln x^2) + \frac{\tan x}{x \ln|x| \ln 5} \right)$$

**Example 10**Find  $\frac{dy}{dx}$  if  $y = \frac{(\cos x)^{\tan 2x} \sqrt{x^5 + 6x}}{\ln(\sec x)}$ 

2 March 1993

**Solution**

$$\ln y = \tan 2x \ln(\cos x) + \frac{1}{2} \ln(x^5 + 6x) - \ln(\ln(\sec x))$$

$$\frac{y'}{y} = 2 \sec^2 x \ln(\cos x) + \tan 2x \cdot \frac{-\sin x}{\cos x} + \frac{5x^4 + 6}{2(x^5 + 6x)} - \frac{1}{\ln(\sec x)} \cdot \frac{\sec x \tan x}{\sec x}$$

$$\frac{dy'}{dy} = \frac{(\cos x)^{\tan 2x} \sqrt{x^5 + 6x}}{\ln(\sec x)} \left( 2 \sec^2 x \ln(\cos x) - \tan 2x \tan x + \frac{5x^4 + 6}{2(x^5 + 6x)} - \frac{\tan x}{\ln(\sec x)} \right)$$

**Example 11**Find  $\frac{dy}{dx}$  if  $\ln|x + e^{xy+2y}| - xy = 2x^2 - y^2$ 

6 March 1997

**Solution**

$$\frac{1}{x + e^{xy+2y}} \cdot [1 + e^{xy+2y}(y + xy' + 2y')] - (y + xy') = 4x - 2yy'$$

**Example 12**IF  $x \ln y - y \ln x^y = 1$  Find  $y'$ **Solution**

$$x \ln y - y^2 \ln x = 1$$

$$\ln y + \frac{x}{y} y' - 2yy' \ln x - \frac{y^2}{x} = 0$$

$$y' \left( \frac{x}{y} - 2y \ln x \right) = \frac{y^2}{x} - \ln y$$

$$y' = \left( \frac{y^2}{x} - \ln y \right) \left( \frac{x}{y} - 2 \ln x \right)^{-1}$$



## Homework

1 Find  $\frac{dy}{dx}$  if  $y = \sqrt[5]{e^{3x^3} \sin x}$  9 October 1998

2 Use logarithmic differentiation to find  $f'(x)$ , where 12 July 2000 A

$$f(x) = \frac{\sqrt[3]{x+1} \sec x}{\sqrt{x} \sin x}$$

3 Find  $\frac{dy}{dx}$  if  $y = \frac{(x + \ln x)(x^5 + 7x)^{10}}{\sqrt{x+11}}$  5 October 1996

4 Find  $y'$  if  $y = \frac{\sqrt[5]{x^3 + 2x} (x^2 + 1)^6}{\sqrt[3]{1-x^3} + \ln|x^4 - e^x|}$  3 Nov. 1994

5 Use logarithmic differentiation to find  $f'(x)$ , where 4 July 1996

$$y = \frac{(5x^4 - 3x)^3 \sec^2 x}{e^{(x^2+1)} \sqrt[3]{1+x^2}}$$

6 Find  $y'$  if  $y = \frac{\sqrt[3]{x+1} \log_5(x^2 + 1)}{(1 + \ln x)^2 e^{1-\sqrt{x}}}$  15 July 2003 A

7 Find  $\frac{dy}{dx}$  if  $y = (1 + x^2)^x$  19 March 2006 A

8 Find  $\frac{dy}{dx}$  if  $ye^{xy} - \ln(1 + x^2y^2) = 3y - 6x^5$  19 March 2006 A

9 Find  $y'$  if  $y = [\log_5(\sqrt{x} + 1)]^{\tan x}$

10 Find  $y'$  if  $y^2 - \ln\left(\frac{x}{y}\right) - 4x = -3$

Homework11Find  $\frac{dy}{dx}$  if  $\log_3|e + y| - 3^{xy} = 6$ 

7 July 1997

